Application and development of inverse theory to Shock Tube problem

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UNIVERSITÉ DE NANTES

outline

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Literature- Survey and conclusions

- Common notion : Inverse problems used with matrix based system and for elliptic PDEs (Heat transfer problems), *rarely used with Fluid Mechanics.*

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- Set up guidelines more like Do's and Dont's for using Inverse Problems with Fluid Systems.

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- Step out of common notion use it for non elliptical and non matrix based systems.
- Set up guidelines more like Do's and Dont's for using Inverse Problems with Fluid Systems.
- Validate use of available inverse problem methods with a case involving non linear Fluid flow phenomenon.

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Problem Statement

Core purpose of this project was to conduct study and research on use of inverse problems with non linear hyperbolic PDE (Euler equation in a Shock Tube) and conduct survey on sensitivity of using inverse methods such systems. Inverse Problem Solution for shock tube

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Inverse Problem Solution for Shock tubes

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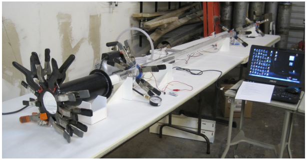
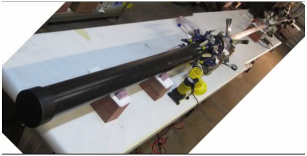
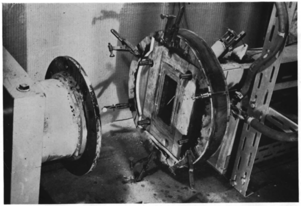
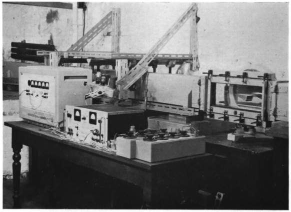
Shock Tube





What is this Shock Tube ?





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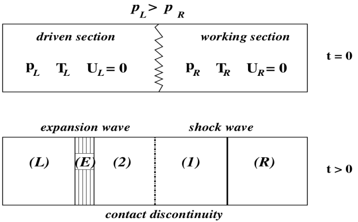
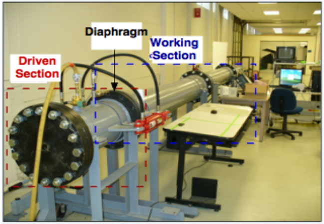
- A device for detonation, transonic, supersonic and hypersonic testing, it was fist invented in France[4,5] (Still used in CNRS IUSTI lab Marseilles).





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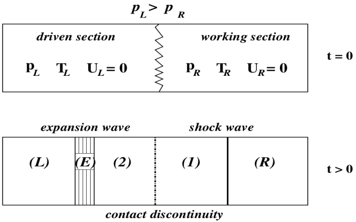

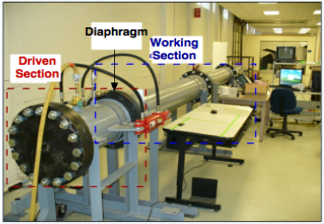

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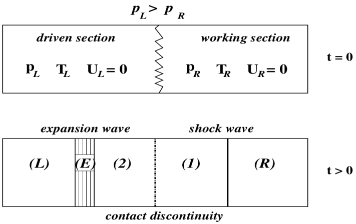
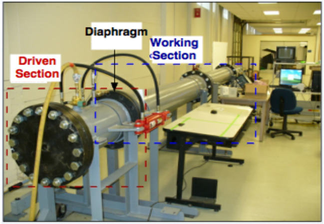
- The time evolution of this problem described by : solving the Euler equations (*Non-linear 1D Hyperbolic PDE*).





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What is this non linear hyperbolic PDE ?

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In matrix form

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho \\ \rho u \\ E\end{pmatrix} + \frac{\partial}{\partial x}\begin{pmatrix}\rho u \\ \rho u^2 + p \\ (E+p)u\end{pmatrix} = 0$$

States and Fluxes

$$U = \begin{pmatrix}\rho \\ \rho u \\ E\end{pmatrix} \quad \text{and} \quad F = \begin{pmatrix}\rho u \\ \rho u^2 + p \\ (E+p)u\end{pmatrix}$$

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Inverse Problem statement

- In practice shock tubes with missing initial conditions [6].

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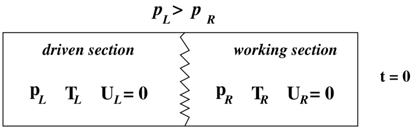

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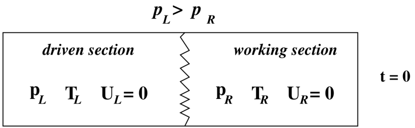

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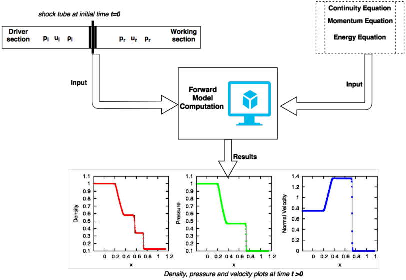

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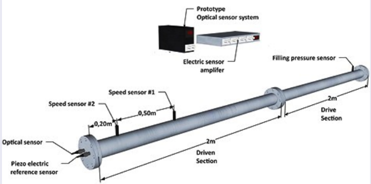

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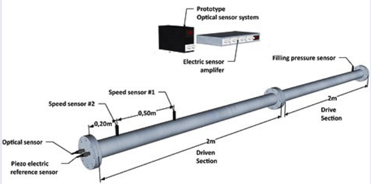

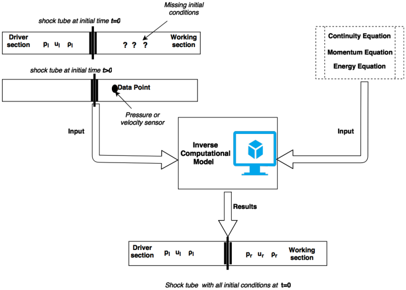

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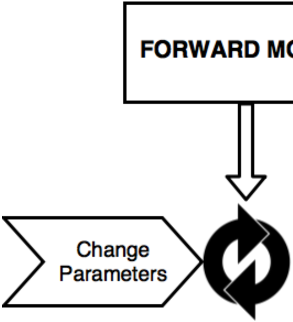
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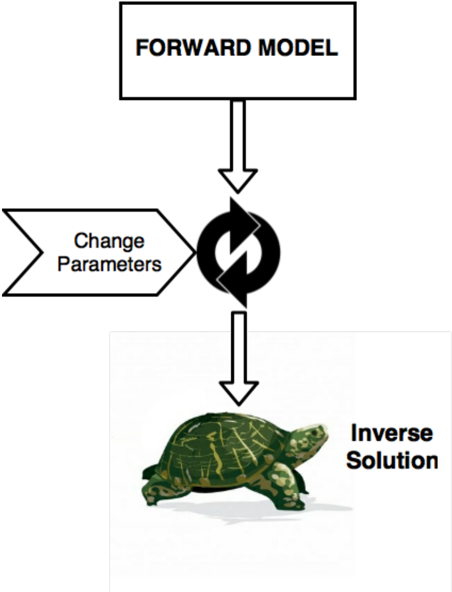
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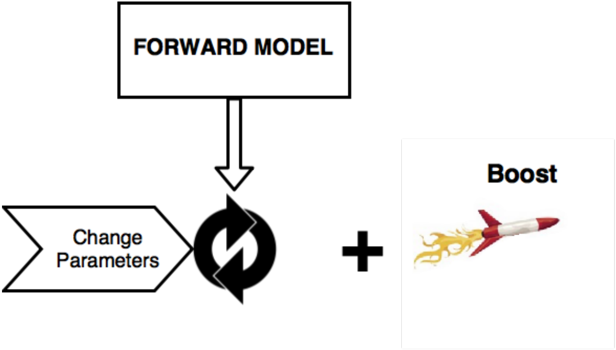
Inverse Solution


FORWARD MODEL
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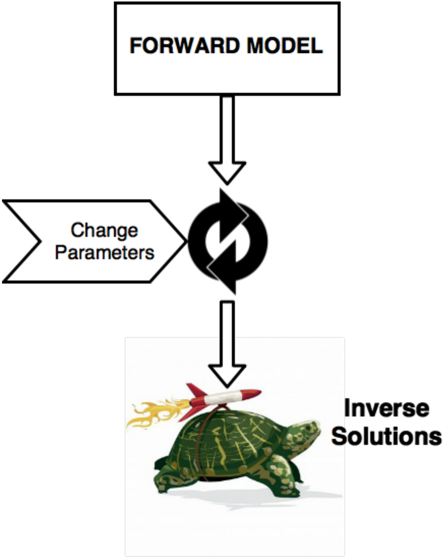
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Inverse Problem Solution for Shock tubes

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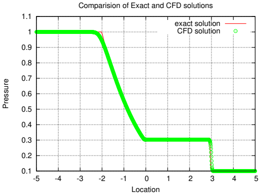

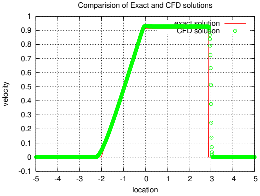

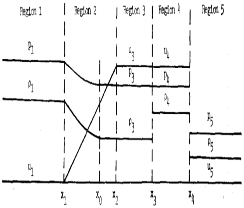

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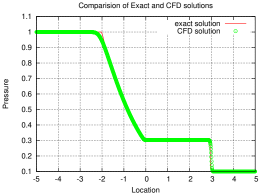

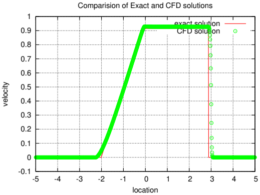

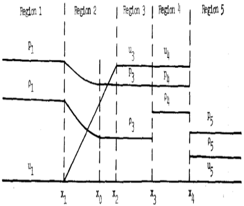

- **Conclusion:** *Scheme is low on errors, capture the flow physics for*

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- Tests reveled
  - ▶ Output pressure p vs changing initial pressure p_r
  - ▶ Output pressure p with changing initial density ρ_r
  - ▶ Output velocity u with changing initial pressure p_r
  - ▶ Output velocity u with changing initial density ρ_r
  - ▶ Output Temperature with changing initial pressure p_r

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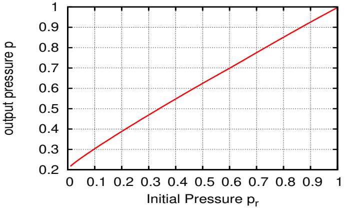

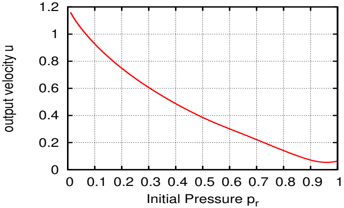

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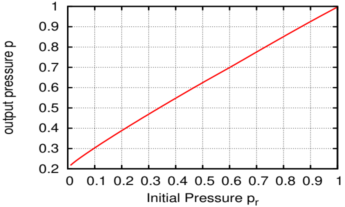

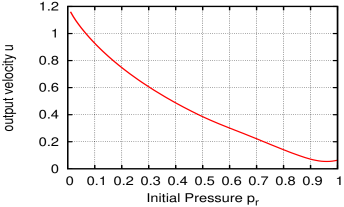

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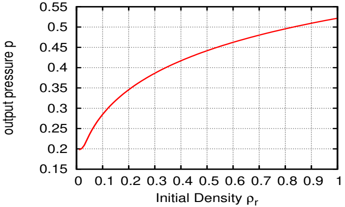

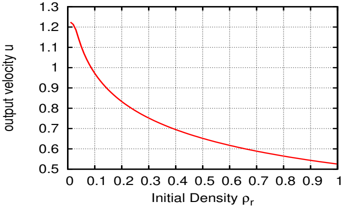

1 Parameter identification

- Results on 1 Parameter identification

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  Pressure identification

Cost Function	Pressure initial	Error %
$\frac{1}{2}(P_{0.5} - P_{exact})^2$	.1112199	11.2199
$\frac{1}{2}(U_{0.5} - U_{exact})^2$	.1112799	11.2799
$\frac{1}{2}(P_{0.5} - P_{exact})^2 + \frac{1}{2}(U_{0.5} - U_{exact})^2$	.1112699	11.2699

POLYTECH

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Cost Function	Density initial	Error %
$\frac{1}{2}(P_{0.5} - P_{exact})^2$	.140109	12.087
$\frac{1}{2}(U_{0.5} - U_{exact})^2$	.140179	12.1439
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POLYTECH

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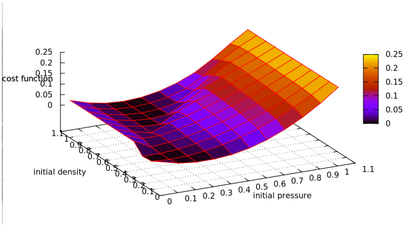
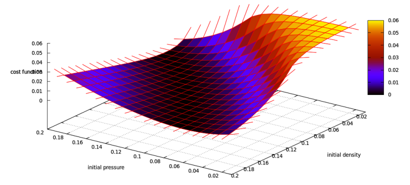
- **Conclusion** *Pressure based cost functions perform better, it will be wise choice to include pressure sensors for further two parameter study.*

POLYTECH

2 Parameter identification- Cost Function plot

- velocity and pressure based pressure based Cost function plot





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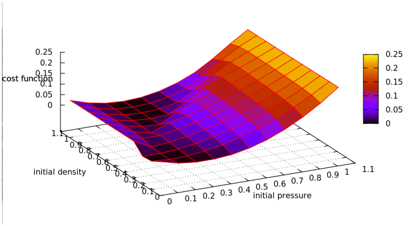
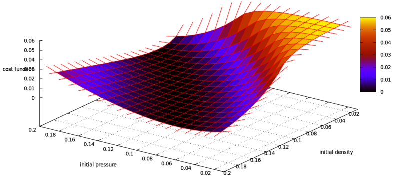
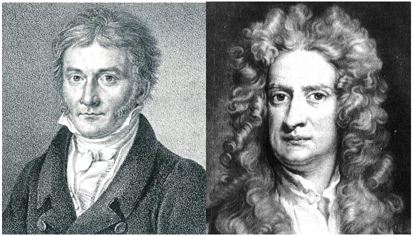




- Minimum approximately exists at desired solution

Gauss-Newton (GN) Algorithm

- Most famous and commonly used method, The method was given by Gauss, It is a modification of Newton's method for finding a minimum of a function.



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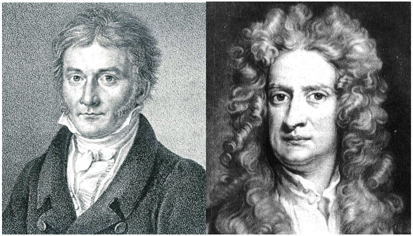

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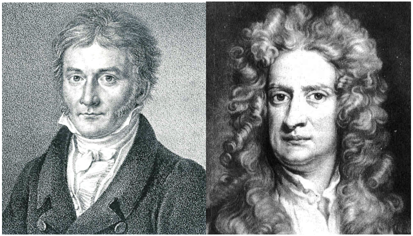

  - Algorithm FAILED.
  - **Conclusion:** *there exists correlation between the initial Pressure and density*
  - *There is a need to apply algorithms which does not deal with Matrices*

Simplex/ Nelder-Mead (NM)

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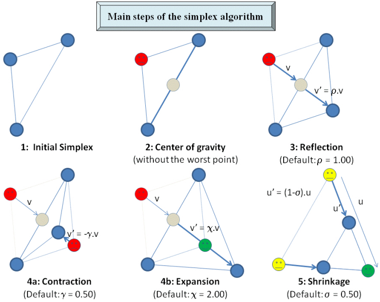

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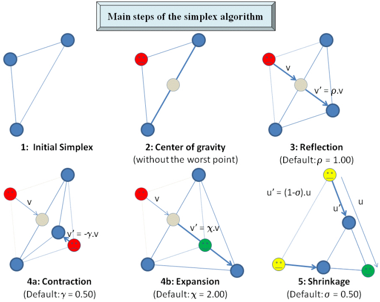

POLYTECH

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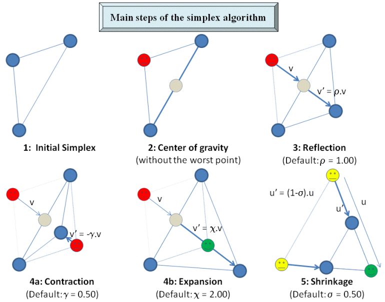

	Iteration	Initial Pressure $\psi_1 = p_r$	Initial Density $\psi_2 = \rho_r$	Error in ψ_1	Error in ψ_2
Case I	16	0.118	0.131	18.1 %	4.8 %
Case II	11	0.112	0.122	12.8 %	2.8 %

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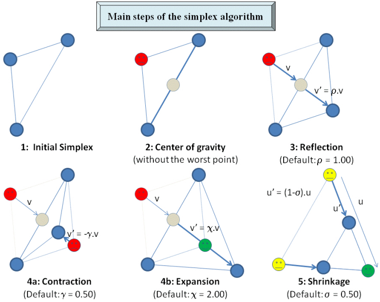

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- **Conclusion** *Algorithm Works well , computationally not much expensive, and is more or less stable.*

POLYTECH

Particle Swarm optimization (PSO) Algorithm

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Particles	Iteration	Initial Pressure $\psi_1 = p_r$	Initial Density $\psi_2 = \rho_r$	Error in ψ_1	Error in ψ_2
10	170	.10631	.13151	6.31 %	4.8 %
20	46	.10633	.13021	6.33 %	4.16 %

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- **conclusion** *Algorithms produces excellent results, it is stable but computationally expensive.*

POLYTECH

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POLYTECH

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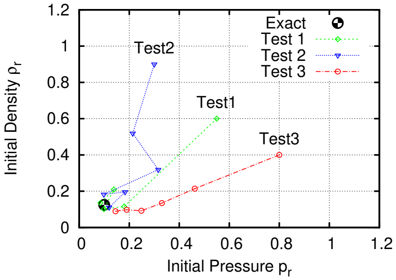

	Iteration	Initial guess ψ^0	Minimum $\overline{\psi}$	Error in $\psi_1(p_r)$	Error in $\psi_2(\rho_r)$
Test 1	4	$[0.55 \;\; 0.6]^T$	$[0.085 \;\; 0.134]^T$	15.0 %	7.2 %
Test 2	5	$[0.30 \;\; 0.9]^T$	$[0.119 \;\; 0.110]^T$	19.0 %	12.2 %
Test 3	5	$[0.8 \;\; 0.4]^T$	$[0.145 \;\; 0.089]^T$	45.6 %	28.8 %

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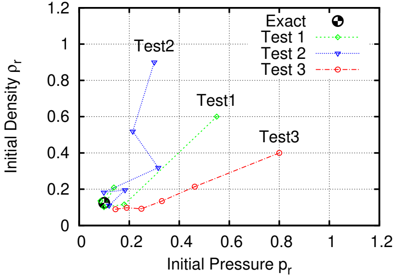

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- **Conclusion:** *Algorithm is not stable, has to be run over and again to know the exact results, it produces higher error than gradient free and it is computationally inexpensive.*

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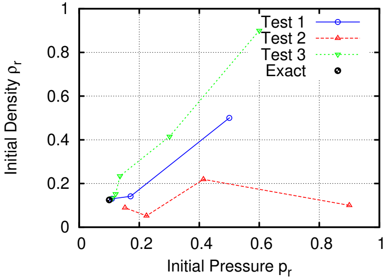

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Test 1	2	$[0.5 \;\; 0.5]^T$	$[0.109 \;\; 0.129]^T$	9.9 %	3.2 %
Test 2	3	$[0.9 \;\; 0.1]^T$	$[0.153 \;\; 0.089]^T$	53.3 %	28.8 %
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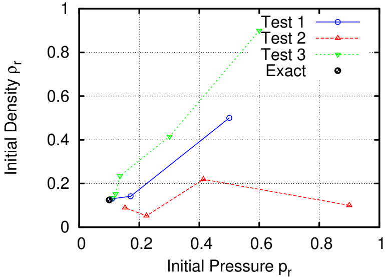

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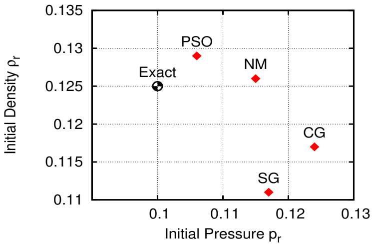
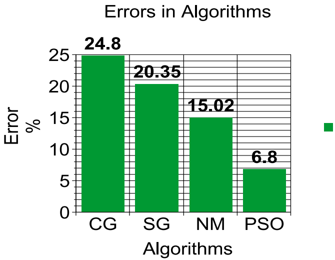
POLYTECH

Conclusion of all algorithms





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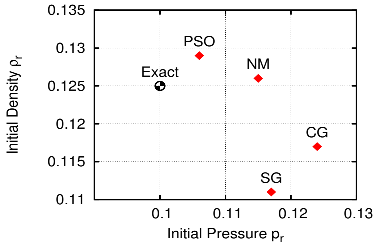

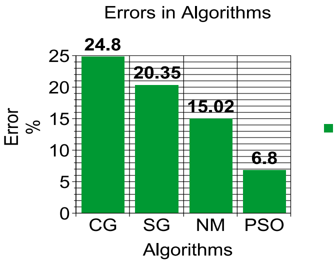

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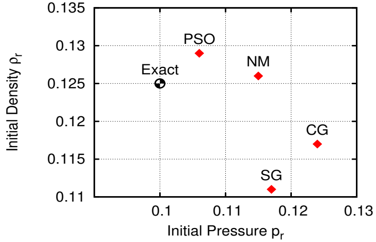

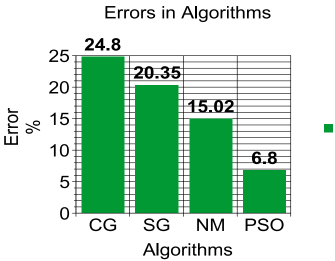

POLYTECH

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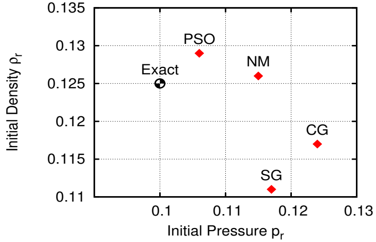

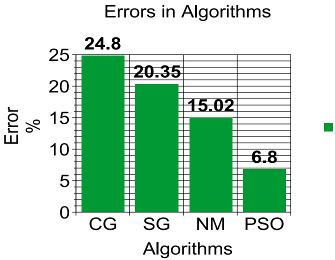

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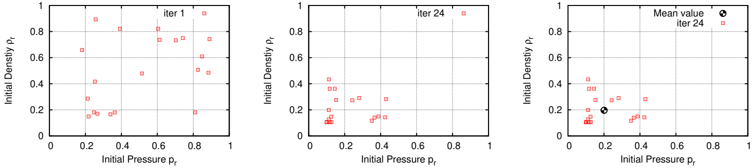

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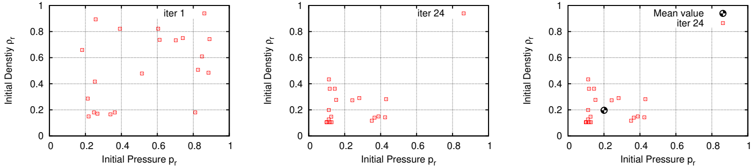

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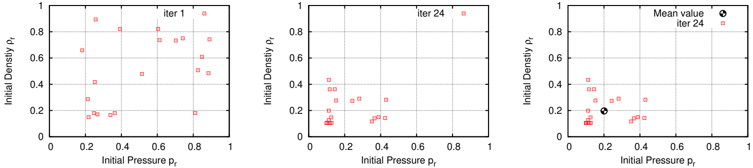

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Iteration (*PSO+CG*)	Initial Pressure $\psi_1 = p_r$	Initial Density $\psi_2 = \rho_r$	Error in ψ_1	Error in ψ_2
24+7=31	.1063	.138	6.3 %	10.4 %

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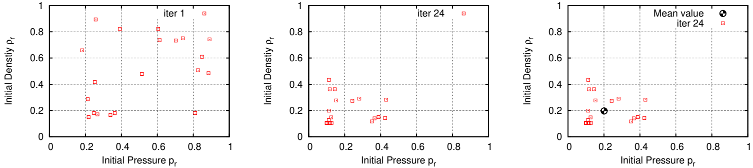

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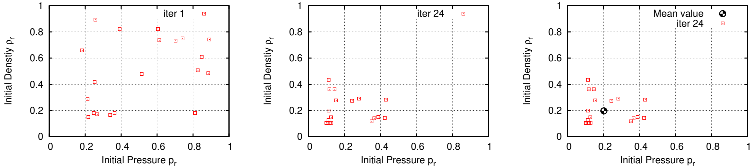

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- *Used with 20 particle PSO saved 22 PSO iterations*, $22 \times 20 = 440$ *forward model iterations.*

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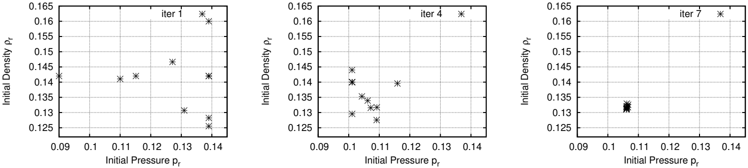

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POLYTECH

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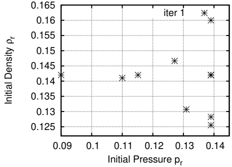

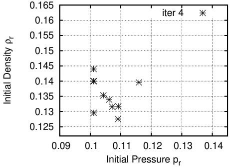

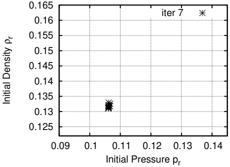

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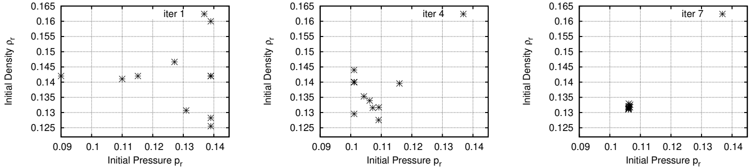

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- **conclusion:** *A stable, low error and computationally economic algorithm. Performed better than PSO1CG2.*
- *Used with 10 particle PSO, saved 163 PSO iterations,* $163 \times 10 = 1630$ *forward model iterations.*

POLYTECH

Project Conclusion

- Inverse shock tube problems has proved counter is possible (non matrix and non elliptic).

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- Certain closures have been presented that may be followed in future to solve other inverse fluid system problems.

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- GN mostly will never perform for static initial conditions, Gradient free are best for inverse fluid mechanics problems.
- Two new algorithms developed PSO1CG2 and PSOCG1 that can be further used.

Thank You For your Attention